

**4th IBERIAN MATHEMATICAL  
MEETING  
(IMM4)**

**October, 5th-7th 2012**

**Valladolid, Spain**



## Foreword

This book contains the program of the 4th Iberian Mathematical Meeting, IMM4, together with the abstracts corresponding to all works presented in it. Keeping the tradition of former editions of the meeting, the Sociedade Portuguesa de Matemática and the Real Sociedad Matemática Española have chosen to focus on three mathematical fields of research, the choices for the present conference being 'Computational Algebra and Applications', 'Mathematics and Life Sciences' and 'Probability and Stochastic Processes'. The collection of talks presented at the conference, including plenary and invited talks, constitute a showcase of the current "state of the art" of mathematical research on these topics in Portugal and Spain.

We would like to thank the Sociedade Portuguesa de Matemática and the Real Sociedad Matemática Española for placing their trust in our team to organize this conference. We hope to live up to this responsibility.

Our gratitude is also due to all the institutions that have provided financial support for this conference, despite the very difficult time of economic crisis that we are living. The Universidad de Valladolid and the Real Sociedad Matemática Española deserve special mention. Without their help, this conference would not have been possible.

Furthermore, we would like also thank all the colleagues at the Institute of Mathematics, IMUVa, at the University of Valladolid that have collaborated in the organization of this conference. They all deserve our thanks and congratulations.

Finally, we welcome the participants at the conference that meet together these days in Valladolid. We wish them all a fruitful and enjoyable stay on both a scientific and personal level, to what hopefully the scientific program and social agenda of the congress will contribute.

Luis M. Abia LLera  
Eustasio del Barrio Tellado  
Miguel Ángel López Marcos

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# Overview





## General Information

### Conference venue

The 4th Iberian Mathematical Meeting will take place at the 'Aulario - Biblioteca', next to the 'Facultad de Ciencias' of the University of Valladolid.

Facultad de Ciencias  
Aulario - Biblioteca  
Campus Miguel Delibes  
Paseo de Belén, 9  
47011 Valladolid

Here is a map of the conference building area (the conference building is marked as 'Aulario').

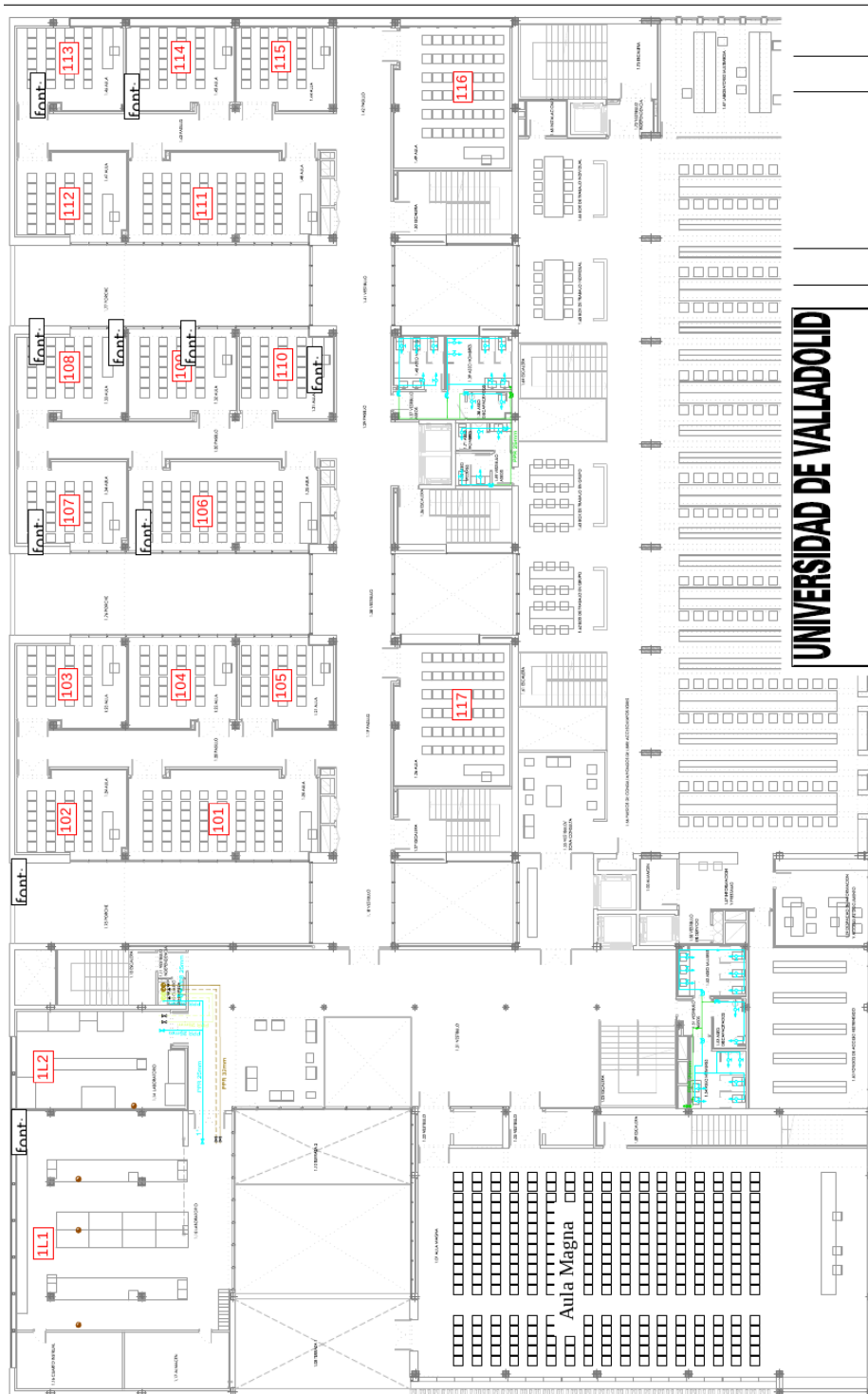


Campus Miguel Delibes, where the conference buildings are located, can be reached by bus. Line 8 stops at Avenida del Valle del Esgueva 57, at the southern access to the campus.

The Miguel Delibes Campus is within walking distance from some of the hotels in the city center (about 30 minutes from Plaza de la Universidad, the site of the historical building of the University of Valladolid).



You can also find here a map of the first floor of the 'Aulario', where talks and coffee breaks will take place.



## Conference Office

The Conference Office is located in front of the Aula Magna.

## Auditoriums and Talks

Plenary talks will take place in the 'Aula Magna' room, on the first floor of the 'Aulario' building. Plenary sessions are labeled P1 to P6 in this program.

Invited sessions will take place in conference rooms 'Aula 101' (sessions CAA1 to CAA4 on Computational Algebra and Applications), 'Aula 106' (sessions MLS1 to MLS4 on Mathematics and Life Sciences) and 'Aula 111' (sessions PSP1 to PSP4 on Probability and Stochastic Processes), also on the first floor of the 'Aulario' building.

Please remember that the time slot assigned for plenary talks is 60 minutes (including a possible short discussion), while for talks in the invited sessions it is 30 minutes. The conference assistants are ready to help the speakers and chairpersons in technical matters.

## Meals and Coffee Breaks

Lunches for conference participants will be served at the meeting. Coffee and a snack will be served during the breaks. Details will be provided at registration.

## Computer Room and Wireless Connection

A computer room with internet connection will be available for conference participants. Details will be provided at registration. Wireless connection through eduroam is likely to be working during the meeting. Unfortunately, the 'Facultad de Ciencias' has moved to this new location very recently and by the time this booklet was printed the wireless connection was not working yet.

## Social Program

**Saturday evening:** *Conference dinner*

The Conference dinner will take place on Saturday evening. Details will be announced in due time.

**Program**





**Session PSP1****Room: 111****18:00-19:30 Invited Session: Probability and Stochastic Processes**

Chair: J. A. Adell

Organizers: C. Matrán and K. F. Turkman

*Controlled branching processes***Inés del Puerto**

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*Non-parametric Bayesian Models in Screening Problems***Marília Antunes**, Sandra Ramos, Antónia Amaral Turkman

Abstract 5

*The empirical cost of optimal incomplete transportation***Eustasio del Barrio**

Abstract 13

**Saturday, October 6th****Session P2****Room: Aula Magna****9:30-10:30 Plenary Speaker: Paula de Oliveira**

Chair: A. Sequeira

*Are mathematical models useful in the treatment of eye pathologies?***Paula de Oliveira**, J.A. Ferreira and Pascoal Silva

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**Session P3****Room: Aula Magna****10:30-11:30 Plenary Speaker: José Antonio Adell**

Chair: C. Matrán

*A differential calculus for linear operators and its applications to limit theorems and information theory***José Antonio Adell**

Abstract 1

11:30-12:00

Coffee Break

**Session CAA2****Room: Aula 101****12:00-13:00 Invited Session: Computational Algebra and Applications**

Chair: J. R. Sendra

Organizers: M. Delgado and L. González Vega

*A functional implementation of rational languages equivalence, using bisimulation***Rogério Reis**, Nelma Moreira

Abstract 29

*Algebraic tools for evolutionary model selection***Marta Casanellas**, Jesús Fernández-Sánchez, Anna M. Kedzierska

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**Session MLS2****Room: Aula 106****12:00-13:00 Invited Session: Mathematics and Life Sciences**

Chair: P. del Oliveira

Organizers: A. Sequeira and M. A. Herrero García

*Cerebral aneurysms and mathematical modeling: from biological investigation to clinical application***Ricardo Pereira**

Abstract 25

*Hybrid  $\tau$ -leaping methods for simulation of stochastic multi-scale cell population models***Tomás Alarcón**, Pilar Guerrero

Abstract 3

**Session PSP2****Room: Aula 111****12:00-13:00 Invited Session: Probability and Stochastic Processes**

Chair: M. Scotto

Organizers: C. Matrán and K. F. Turkman

*Nonlinear time series models - a Bayesian approach to bilinear parameter estimation***P. de Zea Bermudez**, K. F. Turkman and M. A. Amaral Turkman

Abstract 12

*Optimality Criteria for Spatial and Spatio-Temporal Sampling Design based on Entropy***María C. Bueso**, José M. Angulo and Francisco J. Alonso

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13:00-16:00

Lunch

**Session P4****Room: Aula Magna****16:00-17:00 Plenary Speaker: Jorge Almeida**

Chair: P. Carvalho

*Profinite closures of regular languages***Jorge Almeida**

Abstract 4

17:00-17:30

Coffee Break

**Session CAA3****Room: Aula 101****17:30-19:00 Invited Session: Computational Algebra and Applications**

Chair: J. Almeida

Organizers: M. Delgado and L. González Vega

*Optimal Parametrizations of Algebraic Curves and Surfaces***J. Rafael Sendra**

Abstract 33

*Eigenvalue computations in the context of data-sparse approximations of integral operators***Paulo B. Vasconcelos**, J. E. Roman, A. N. Nunes

Abstract 36

*Provable security: why should mathematicians care***María Isabel González Vasco**

Abstract 18

**Session MLS3****Room: Aula 106****17:30-19:00 Invited Session: Mathematics and Life Sciences**

Chair: T. Alarcón

Organizers: A. Sequeira and M. A. Herrero García

*Haemodynamics in Cerebral Aneurysms***Alberto Gambaruto**

Abstract 17

*Modeling multicellular growing systems***J. Nieto**, A. Bellouquid, N. Bellomo and J. Soler

Abstract 23

*Mathematical model of eye pathologies related with the porosity of tissues***Pascoal Silva**, J.A. Ferreira and Paula de Oliveira

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**Session PSP3****Room: Aula 111****17:30-19:00 Invited Session: Probability and Stochastic Processes**

Chair: I. del Puerto

Organizers: C. Matrán and K. F. Turkman

*Characterizing losses during busy-periods in finite buffer oscillating queues***Fátima Ferreira**, António Pacheco and Helena Ribeiro

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*Efficient management of dairy farms***Aureli Alabert**

Abstract 2

*Functional data analysis applied to environmental chlorophyll bio monitoring***Raquel Menezes**, A.Castro, J.Arruda-Neto, J.Harari, M.Febrero-Bande

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**Sunday, October 7th****Session P5****Room: Aula Magna****9:30-10:30 Plenary Speaker: Juan Soler**

Chair: M. Herrero

*Pattern formation in a flux limited reaction-diffusion equation of porous media type modeling morphogenetic action***Juan Soler**

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**Session P6****Room: Aula Magna****10:30-11:30 Plenary Speaker: Antonio Pacheco**

Chair: E. del Barrio

*Level Crossing Ordering of Markov and Semi-Markov Processes***Antonio Pacheco** and Fátima Ferreira

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11:30-12:00

Coffee Break



**Session CAA4****Room: Aula 101****12:00-13:00 Invited Session: Computational Algebra and Applications**

Chair: P. Giménez

Organizers: M. Delgado and L. González Vega

*Gröbner Bases and Oriented Matroids***Paula Carvalho**

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*Integral bases in global fields***Jesús Montes**, Jordi Guàrdia and Enric Nart

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**Session MLS4****Room: Aula 106****12:00-13:00 Invited Session: Mathematics and Life Sciences**

Chair: J. Soler

Organizers: A. Sequeira and M. A. Herrero García

*New absorbing conditions for FSI problems in blood flow***J. Janela**, A. Moura, A. Sequeira

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*Food processing by high pressure technologies. A mathematical approach***Angel M. Ramos**

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**Session PSP4****Room: Aula 111****12:00-13:00 Invited Session: Probability and Stochastic Processes**

Chair: A. Pacheco

Organizers: C. Matrán and K. F. Turkman

*Integer-valued Asymmetric Power ARCH Processes***Manuel Scotto**, Conceição Costa and Isabel Pereira

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*Study of the density for semilinear dissipative parabolic SPDEs***Lluís Quer-Sardanyons**

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Closing Session



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# **Abstracts**



# 1 A differential calculus for linear operators and its applications to limit theorems and information theory

José A. Adell

Universidad de Zaragoza

**Keywords:** linear operator, differential calculus, Poisson approximation, Poisson mixture, Edgeworth expansion, subordinator, entropy, Poisson channel.

We introduce a differential calculus for linear operators represented by a family of finite signed measures, in particular, by stochastic processes. Such a calculus is based on the notions of  $g$ -derived operators and processes and  $g$ -integrating measures,  $g$  being a right-continuous nondecreasing function. Depending on the choice of  $g$ , this differential calculus works for non-smooth functions and under weak integrability conditions. For linear operators represented by stochastic processes, we provide a characterization criterion of  $g$ -differentiability in terms of characteristic functions of the random variables involved. Various illustrative examples are considered.

Three kinds of applications will be discussed: In first place, we give sharp estimates in total variation, as well as certain types of stop-loss metrics, in signed Poisson approximation of Poisson mixtures, paying special attention to the case of the negative binomial distribution. The problem of the best Poisson approximation will also be sketched.

In second place, we obtain Edgeworth expansions for  $E\phi(Z_t) - E\phi(Z)$ , where  $(Z_t)_{t \geq 1}$  is a standardized subordinator,  $Z$  is a standard normal random variable, and  $\phi$  is a suitable smooth function. We provide explicit upper bounds for the remainders, thus getting rid off the 'big or little o' terms. We also give some simple sufficient conditions on  $\phi$  to ensure the property of monotonic convergence for  $E\phi(Z_t)$ .

Finally, we derive upper and lower bounds for the entropy of some familiar discrete distributions, such as the Poisson and the binomial distributions, that are asymptotically tight and easy to compute in terms of their centered moments. Also, we obtain explicit upper and lower bounds for the capacity of the discrete-time Poisson channel, subject to an average intensity constraint. These bounds are sharp both at infinity and at the origin.

## References

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Session P3

# 2 Efficient management of dairy farms

Aureli Alabert

Universitat Autònoma de Barcelona

**Keywords:** stochastic simulation, optimisation

A typical dairy farm hosts a certain number of cows, that are supposed to produce as much milk as possible. For the cows to produce milk, they have to get pregnant and give birth. There is a typical curve of milk production after giving birth, but there are several parameters that are random and depend on each particular animal.

Every week the farmer must decide whether to send non-profitable cows to the slaughterhouse, obtaining some revenue for the meat, and replace them with young new cows. This decision is usually taken on intuitive grounds, and therefore an economic model is sought to help optimise the overall profit of the farm.

This is a joint ongoing project with the Service of Animal Nutrition and Welfare in the UAB, and constitutes an example of application of stochastic simulation to an optimisation industrial problem.

Session PSP3

# 3 Hybrid $\tau$ -leaping methods for simulation of stochastic multi-scale cell population models

Tomás Alarcón<sup>1</sup>, Pilar Guerrero<sup>1</sup>

Centre de Recerca Matemàtica, Bellaterra, Barcelona

**Keywords:**

We show how complex multi-scale models of cell populations can be formulated as stochastic age-dependent Markov processes described by an age-dependent Master Equation. We then move on to establishing the numerical methodology to study such systems. In particular we propose a generalisation of the classical  $\tau$ -leaping method which accounts for the multi-scale nature of our problem. Furthermore, we propose an asymptotic perturbation method, which generalises the WKB method for the Master Equation to our age-dependent model.

Session *MLS2*

## 4 Profinite closures of regular languages

**Jorge Almeida**

Universidade do Porto

**Keywords:** finite and profinite semigroups, regular languages, profinite topologies, topological closure

A finite automaton is a very simple but very useful computational model with wide applications in particular in Computer Science. Languages recognized by finite automata are said to be *regular*. The most natural algebraic structure associated with a finite automaton is its transition semigroup. Based on this connection, Samuel Eilenberg formulated in 1976 a natural framework for linking the classification of regular languages with that of finite semigroups. Some (but not all) natural problems on classes of regular languages can be thus translated into problems on certain classes of finite semigroups. The latter are called *pseudovarieties* and the problems often concern the decidability of their membership.

Free semigroups relative to pseudovarieties are in general infinite and do not characterize them. Yet, their profinite completions, which can be viewed as Stone duals of the corresponding Boolean algebras of regular languages, serve that purpose. Profinite techniques play therefore a key role in the ensuing theory of finite semigroups.

The particular problem of concern in this talk is the following: given two regular languages  $K$  and  $L$  and a pseudovariety  $V$ , can  $K$  and  $L$  be separated by a  $V$ -language? In topological terms, are the pro- $V$  closures of  $K$  and  $L$  disjoint? In the case of the pseudovariety

$G$  of all finite groups, the decidability of this problem is equivalent to an algebraic decision problem raised in the late nineteen sixties by J. Rhodes and which came to be known as the “Type II Conjecture”. The strong form of the conjecture translates into not only the decidability of our problem but also into a concrete effective solution. The key of the solution is a procedure proposed by Pin and Reutenauer in 1991 for the description of the pro- $G$  closure of a regular language which consists in replacing in a regular expression for the language the Kleene star (i.e., submonoid generation) by subgroup generation. The correctness of this procedure was reduced by Pin and Reutenauer to a property of the profinite topology of the free group which was proved by Ribes and Zalesskiĭ in 1993. An alternative, independent and more general proof of the Type II Conjecture was given by Ash in 1991, through a result which was rediscovered in 1997 by Herwig and Lascar in the guise of a theorem in Model Theory.

The purpose of this talk is to show that the Pin-Reutenauer procedure holds more generally for many pseudovarieties  $V$ . In fact, it turns out to be equivalent to a simple topological property of certain  $V$ -free algebras.

This talk is based on recent and ongoing joint work with J. C. Costa (Universidade do Minho), O. Klíma (Masaryk University), and M. Zeitoun (Université de Bordeaux 1).

## References

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- [2] Rhodes, J. and Steinberg, B. (2009). *The  $q$ -theory of Finite Semigroups*, Springer.

Session *P4*

## 5 Non-parametric Bayesian Models in Screening Problems

**Marília Antunes**<sup>1</sup>, Sandra Ramos<sup>2</sup>, Antónia Amaral Turkman<sup>1</sup>

<sup>1</sup> Faculty of Sciences and Center of Statistics and Applications, University of Lisbon

<sup>2</sup> ISEP and Center of Statistics and Applications, University of Lisbon

**Keywords:** screening, non-parametric Bayesian models, MCMC methods

Screening methods are used to decide if an individual is considered a success (the response variable  $Y$  belongs to a known region  $C_Y$ ) or not, based on information given through a  $d$ -dimensional characteristic vector  $\mathbf{X}$ , supposed to be informative about  $Y$ . More specifically, the screening procedure consists in building a specification region  $C_X$  in the  $d$ -dimensional space such that a future individual with a characteristic vector in  $C_X$  has higher probability of being a success. In the screening procedure established in [1],  $C_X$  is obtained considering an optimality criteria based on the maximization of  $P(Y \in C_Y | \mathbf{X} \in C_X; \mathcal{D})$  constrained to the class of regions  $C_X$  of size  $\alpha$ , that is, with predictive probability of screening  $\alpha$ , fixed. The optimal specification region verifying the above definition is

$$C_{\mathbf{X}} = \{\mathbf{x} \in \mathbb{R}^d : P(Y \in C_Y | \mathbf{x}; \mathcal{D}) \geq k\},$$

where  $k$  is such that  $P(\mathbf{X} \in C_{\mathbf{X}} | \mathcal{D}) = \alpha$ .

Usually, both in the classical and the Bayesian approaches, a parametric model is considered for  $(Y, \mathbf{X})$ . Such models often imply the specification of a certain number of assumptions which are difficult to verify in practice and such that if disregarded, misleading results can be obtained. In this work we present semi-parametric and non-parametric Bayesian solutions for the screening problem. The proposed approaches are illustrated with some applications in problems of unsupervised classification.

## References

[1] Turkman K. F., Amaral Turkman, M. A. (1989). Optimal screening methods. *J. Royal Statist. Soc. B*, 51, 287–295.

Session PSP1

## 6 Non fickian diffusion models for highly diffusive tumors

João Ricardo Branco, J.A. Ferreira and P. de Oliveira

Universidade de Coimbra

### Keywords:

Gliomas are diffusive and highly invasive brain tumors. Even with aggressive surgical resection and radiotherapy and/or chemotherapy, gliomas almost always recur, with fatal consequences. The median survival for patients with glioma doesn't go beyond one

year. Thus, due to their highly invasive and recurrent behaviour, effective therapeutic strategies for gliomas are extremely important to improve survival time.

Although more clinical trials are necessary to determine the optimal treatment strategies, the development of mathematical models to address these questions is also appropriate and timely. Carefully devised and validated mathematical models might be useful for developing hypotheses to be tested in future clinical trials, and for optimizing the design of future trials.

The aim of this talk is to present a mathematical model for gliomas growth, characterized by an integro-differential equation with a certain memory effect, and to conclude some sufficient conditions to be verified when chemotherapy or radiation are used.

Session MLS1

## 7 Optimality Criteria for Spatial and Spatio-Temporal Sampling Design based on Entropy

María C. Bueso<sup>1</sup>, José M. Angulo<sup>2</sup> and Francisco J. Alonso<sup>2</sup>

<sup>1</sup> Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Murcia, Spain

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**Keywords:** mutual information, optimality criterion, Shannon's entropy, spatial process, spatio-temporal process

During the last two decades, applications in environmental sciences have shown an increasing interest in designing effective sampling strategies for estimating stochastic processes defined in a spatial and spatio-temporal context. Depending on the sampling objectives to be achieved and under different settings, a diversity of approaches have been introduced in the related literature (see, for example, an overview in [5] and [6]). In this talk, we present an entropy-based formulation of the problem in a state-space framework, defining as optimality criterion to maximize the mutual information between the potentially observable variables in a spatial or spatio-temporal configuration to be selected from a set of candidates and the variables of interest. After presenting the methodology in the purely spatial case, we introduce a time-adaptive sampling procedure where the configuration of the spatial locations can be modified at each time depending on the spatio-temporal evolution of the observed pro-

cess. The methodologies introduced in the different contexts are illustrated with simulated cases in which the dependence structure of the underlying process is characterized in terms of a parametric model, where the influence on the resulting networks of the assumed model is analyzed.

**Acknowledgments:** This work has been supported in part by project P08-FQM-3834 of the Andalusian CICE, and project MTM2009-13250 of the SGPI, MICINN, Spain.

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- [3] Angulo, J.M., Ruiz-Medina, M.D., Alonso, F.J. and Bueso, M.C. (2005). Generalized approaches to spatial sampling design, *Environmetrics*, **16**, 523–534.
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Session PSP2

## 8 Gröbner Bases and Oriented Matroids

Paula Carvalho<sup>1</sup>

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**Keywords:** Graßmann-Plücker polynomials, Gröbner Basis

Given an orientable, two dimensional, simplicial complex without boundary, as a (closed, orientable) surface we know it can be embedded in the euclidean space of dimension three. Yet, in general, the embedding does not preserve the complex, in the sense that the image of the edges are no longer always straight, and hence the image of the triangles are also no longer

always triangles. Whether such a *geometric* (vs. *topological*) embedding exists is an important problem, with very few answers, even for small dimensions.

In [1], it was developed for oriented matroids the typical topological tools of *intersection* and *linking numbers*. With these tools we can prove the non-existence of oriented matroids admissible with a given complex.

One advantage of this algebraic definition is the possibility of handling "automatic proofs". The non-existence of oriented matroids admissible with a given complex can be done (with a computer) by showing that a certain polynomial is a member of the ideal of the uniform chirotope variety – that is the ideal generated in  $GF_3[A(n, r)]$  by the *three-term Graßmann-Plücker polynomials* together with the polynomials of the form  $1 - [\lambda]^2$ .

The membership problem has a quit simple answer if it is known a Gröbner base for the required ideal but although the process and the algorithm are simple in nature, it is a well-known fact that the problem of constructing such a base is an intrinsically hard problem. Introducing the new notion of co-polynomials [2], that are polynomials arising from the Graßmann-Plücker polynomials, it is possible to characterize a base for a small number os points and to reduce considerably the process as to space and time consume.

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Session CAA4

## 9 Algebraic tools for evolutionary model selection

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**Keywords:** algebraic statistics, evolutionary model selection, phylogenetic invariants

The selection of an evolutionary model to best fit given molecular data is usually a heuristic choice. In his seminal book, biologist J. Felsenstein suggested that

certain linear equations satisfied by the expected probabilities of patterns observed at the terminal species of a phylogenetic tree could be used for model selection. It remained an open question, however, whether these equations were sufficient to fully characterize the evolutionary model under consideration.

In this talk, we will introduce the use of algebraic varieties in evolutionary biology and present a solution to the problem above. Namely, we will see how a linear space completely characterizes the space of distributions arising from mixtures of phylogenetic trees. As a byproduct we will present a novel model selection algorithm and we will compare its performance to usual model selection methods. This work can be found in [1] and [2].

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Session CAA2

## 10 Quantitative equidistribution for the solutions of a system of sparse polynomial equations

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**Keywords:** Equidistribution, zeros of systems of polynomials, exponential sums, sparse resultants, random systems of polynomials.

For a system of Laurent polynomials  $f_1, \dots, f_n \in \mathbb{C}[x_1^{\pm 1}, \dots, x_n^{\pm 1}]$  whose coefficients are not too big with respect to its facet resultants, we show that the solutions in the algebraic torus  $(\mathbb{C}^\times)^n$  of the system of equations  $f_1 = \dots = f_n = 0$ , are approximately equidistributed near the unit polycircle. This generalizes to the multivariate case, up to an exponent, a classical result due to Erdős and Turán on the distribution of the arguments of the roots of a univariate polynomial.

We apply this result to bound the number of real roots of a system of Laurent polynomials, and to study

the asymptotic distribution of the roots of systems of Laurent polynomials over  $\mathbb{Z}$  and of random systems of Laurent polynomials over  $\mathbb{C}$ .

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Session CAA1

## 11 Are mathematical models useful in the treatment of eye pathologies?

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**Keywords:**

The front part of the eye is filled with a clear fluid called aqueous humor (AH) that is composed mostly of water and has two main roles: to deliver oxygen and nutrition to tissues within the eye, and to maintain a correct pressure balance in the anterior chamber. The fluid is continuously produced and it leaves the eye through the trabecular mesh. In some situations it occurs an obstruction of the trabecular mesh, which narrows over time. An increase of the pressure inside the anterior chamber of the eye, the so called intraocular pressure, is then observed. In other situations there exists an excessive production of aqueous humor and the fact can also affect the intraocular pressure (IOP). This buildup of IOP is by far the most common risk factor of glaucoma because more likely the optic nerve will be damaged.

In this talk we address the dynamics of A.H in healthy and pathological situations. We present a mathematical model which simulates drug delivery through the cornea and its effects on IOP. The model consists of three coupled systems of Partial Differential Equations linked by interface conditions: drug delivery from a therapeutic lens; diffusion and metabolic consumption in the cornea; diffusion, convection and metabolic consumption in the anterior chamber of the eye. Different causes of IOP increase and its treatments will be simulated. The effects of several type of drugs which act in the inflow or in the outflow of AH from the anterior chamber will be considered.

The information that mathematical models of aqueous humor dynamics can give to clinicians will be discussed.

Session P2

## 12 Nonlinear time series models - a Bayesian approach to bilinear parameter estimation

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**Keywords:** Nonlinear time series models, Bilinear models, Bayesian statistics, MCMC

Nonlinear time series models have become very popular as alternatives to the traditional linear approaches. Many real time series exhibit nonlinear behavior, such as the occurrence of sudden “jumps” and the appearance of asymmetrical cycles. These features are commonly observed in areas such as finance, economy and seismology. The class of bilinear models (Subba Rao and Gabr [2]) is particularly useful for representing time series that may generate sudden large bursts. They are the most general class of nonlinear models and are important for several reasons, such as the fact that they are a generalization of the ARMA models. A bilinear model of order  $(p, q, r, s)$  is defined as

$$Y_t = \sum_{i=1}^p a_i Y_{t-i} + \sum_{j=1}^q b_j \varepsilon_{t-j} + \sum_{i=1}^r \sum_{j=1}^s c_{ij} Y_{t-i} \varepsilon_{t-j} + \varepsilon_t,$$

where  $\{\varepsilon_t\}$  is a sequence of iid random variables with zero mean and variance  $\sigma^2$ . In spite of their interest, they are very difficult to work with in practice. The estimation of the model parameters is a problematic issue, namely due to the strong constraints in the parameter space imposed by the invertibility and stationarity conditions (Turkman et al [3]).

The particle Markov chain Monte Carlo (PMCMC) is a recent methodology that aims to overcome some of the problems observed with Markov chain Monte Carlo (MCMC) algorithms. It has recently been proposed by Andrieu et al. [1] and its utility has been shown in several complex settings such as the state space models.

Let  $Y_t$  be a bilinear process of order  $(1,0,1,1)$  given by  $Y_t = aY_{t-1} + bY_{t-1}\varepsilon_{t-1} + \varepsilon_t$ . Let us consider that

the observable process  $Y_t$  is expressed as  $Y_t = X_t + \varepsilon_t$ , where  $X_t$  is a Markov process. In order to guarantee invertibility and stationarity, the parameters  $a$ ,  $b$  and  $\sigma^2$  must satisfy the condition  $a^2 + b^2\sigma^2 < 1$ . In this framework, interest focuses on showing the benefits of using the PMCMC methodology for obtaining the marginal posterior distribution of  $\theta = (a, b, \sigma^2)$ . We will apply the particle marginal Metropolis-Hastings sampler to first order bilinear models. The ultimate objective is to extend such results to higher order bilinear models.

**Acknowledgements:** The research is (partially) supported by the projects PEst-OE/MAT/UI0006/2011 and PTDC/MAT/118335/2010.

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Session PSP2

## 13 The empirical cost of optimal incomplete transportation

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**Keywords:** optimal transportation, optimal matching, optimal incomplete transportation, optimal partial matching, random quantization, rates of convergence.

We consider the problem of optimal incomplete transportation between the empirical measure on an i.i.d. uniform sample on the  $d$ -dimensional unit cube,  $[0, 1]^d$ , and the true measure. This is a family of problems lying in between classical optimal transportation and nearest neighbor problems. We show that the empirical cost of optimal incomplete transportation vanishes at rate  $O_P(n^{-1/d})$ , where  $n$  denotes the sample size. In dimension  $d \geq 3$  the rate is the same as in classical optimal transportation, but in low dimension it is (much) higher than the classical rate.

Session PSP1



## 14 Controlled branching processes

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**Keywords:** branching processes, extinction problem, asymptotic behaviour, method of the moment, maximum likelihood estimation, weighted conditional least squares estimation.

The behaviour of the long-time evolution of many populations depends on the conditions on the offspring reproduction model and on the span life of their individuals. The theory of branching processes was developed to deal with these models. Initially it was motivated to explain the extinction phenomenon of certain family lines of European aristocracy and names like Bienaymé, Galton and Watson are linked to the former studies. Nowadays, this kind of processes are treated extensively for their mathematical interest and as theoretical approaches to solving problems in applied fields such as Biology, Epidemiology, Genetics, and Cell Kinetics (gene amplification, clonal resistance theory of cancer cells, polymerase chain reactions, the evolution of infectious diseases, sex-linked genes, stem cells, etc.), Computer Algorithms (the spread of scanning worm on the internet) and Economics (stock prices, barrier option pricing, etc), and, of course, Population Dynamics, to mention only some of the more important applications.

In particular, in this talk we are interested in the class of controlled branching processes. Adopting the terminology of the population dynamics, these are discrete-time stochastic processes that are used to describe the evolution of populations where each individual lives for one unit of time and is replaced by a random number of offspring and a control in the population size in each generation is needed. This control is made by determining through a random mechanism the number of individuals with reproductive capacity at each generation. We present a survey of the main results obtained until now about the extinction problem, the limit behaviour and the statistical inference arising from this model. These results have been obtained in collaboration with several coauthors: Miguel González, Cristina Gutiérrez, Rodrigo Martínez, Manuel Molina, Manuel Mota, Alfonso Ramos, all of them from University of Extremadura, and T.N. Sriram and A. Bhat-tacharya from University of Georgia, USA.

**Acknowledgements:** This research was supported by the Ministerio de Economía y Competitividad, the Junta de Extremadura and the FEDER through the grants MTM2009-13248 and GR10118.

*Session PSP1*

## 15 Techniques from the classification of nilpotent semigroups

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**Keywords:** nilpotent semigroup, classification, constraint programming, coclass

A semigroup  $S$  is a set together with an associative binary operation. We call  $S$  *nilpotent* if it contains a zero element  $z$  and  $S^{c+1} = \{s_1 \cdots s_{c+1} \mid s_i \in S\} = \{z\}$  for some  $c \in \mathbb{N}_0$ . The smallest such  $c$  is the *nilpotency class* of  $S$ .

In my talk I will present a variety of techniques applied in the classification of finite nilpotent semigroups. First, the determination of nilpotent semigroups and their automorphism groups by computer search performed using computer algebra and constraint satisfaction; further, the incorporation of nilpotent semigroups of order at most 8 into a data library; and finally an algorithmic approach to the classification using a new parameter for nilpotent semigroups called *nilpotency coclass*, which for a finite nilpotent semigroup  $S$  of nilpotency class  $c$  equals  $|S| - c - 1$ .

The talk will concentrate on general ideas rather than results for nilpotent semigroups.

*Session CAA1*

## 16 Characterizing losses during busy-periods in finite buffer oscillating queues

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**Keywords:** Batch arrivals, busy-periods, customer losses, oscillating queues

This work addresses loss characteristics during busy-periods of oscillating  $M^X/G/1/(n, a, b)$  queues, proposed in the literature with the aim of improving the performance of standard queueing systems, as the latter are not able to achieve higher rates of service utilization concurrently with small loss probabilities. These are finite single server queues, with buffer size  $n$ , having compound-Poisson input and general distributed service times, which alternate between two sets of parameters according to the evolution of the number of customers in the system and two thresholds,  $a$  and  $b$ . Due the finite capacity of these systems, customers may have to be blocked at arrival due to overflow in congestion periods. In particular, whenever upon arrival a batch does not find enough space in the buffer to fully accommodate the customers of the batch, the buffer is filled up with the customers in the front part of the batch and the remaining customers of the batch, for whom there is no space available in the buffer, are blocked. As a consequence, the number of customer losses taking place in the queue, and in particular taking place in their busy-periods, is an important descriptor to evaluate the performance of these queues and is the focus of this work. In particular, an efficient recursive procedure is presented to evaluate the probability mass function of the number of losses during busy-periods started with an arbitrary number of customers in system. The effectiveness of this computational procedure is illustrated comparing the loss characteristics of  $M^X/G/1/(n, a, b)$  queues with different capacities, batch size and service time distributions, and arrival and service parameters.

Session PSP3

## 17 Haemodynamics in Cerebral Aneurysms

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**Keywords:** blood flow, vortex, flow structure, critical point

Haemodynamic induced stresses and transport phenomena are widely correlated to a comprehensive range of cardiovascular diseases, both at macro- and micro-scales. Aneurysms, especially cerebral, have benefited significant attention from the applied mathematics, engineering and computer science communities, which have provided means of delivering high resolution computational simulations of blood flow based on patient-specific setup acquired from non-invasive medical imaging. While the mathematical modelling of the human physiological system is under development, with a drive to include the haemodynamics, structural mechanics, biochemical cascades and further medical factors, the current possible clinical application relies on correlation of observed phenomena to disease occurrence.

In this work means to describe and characterise the flow field in the free-slip and no-slip domains are discussed in the context of cerebral aneurysms, reconstructed from in vivo medical imaging. The approaches rely on a Taylor series expansion of the velocity field to first order terms that leads to a system of ODEs, the solution to which locally describes the motion of the flow. On performing the expansion on the vessel wall using the wall shear stress, the critical points can be identified and the near-wall flow field parallel to the wall can be concisely described and visualised. Furthermore the near-wall expansion can be expressed in terms of relative motion, and the near-wall convective transport normal and parallel to the wall can be accurately derived on the no-slip domain. Together, these approaches give a viable and robust means to identify and describe fluid mechanic phenomena both qualitatively and quantitatively, leading to feasible practical use in biomedical applications.

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Session MLS3

## 18 Provable security: why should mathematicians care

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**Keywords:** cryptographic scheme, public key, formal security proof

Since the late 70s, the computational hardness of certain mathematical problems has been applied to develop cryptographic tools. At first, most of these designs have been introduced at a somewhat conceptual level, and soon relevant hardships for developing practical schemes from them were identified. Many of the security problems that arise, come in the absence formal security proofs. In this talk, we will briefly introduce some of the established provable security notions for encryption schemes (see, for instance [3]) and give examples to highlight the reasons why some practical attacks work. Some of them will be drawn from schemes that tried to exploit the hardness of group theoretical problems for cryptography, for a recent survey on the topic see [1]. To illustrate the usefulness of developing a suitable theoretic framework, we consider a setting in which parties use a public key encryption scheme and signature scheme with a single public key/private key pair—so the private key  $sk$  is used for both signing and decrypting. Such a simultaneous use of a key is in general considered poor cryptographic practice, but from an efficiency point of view looks attractive (see [2]).

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Session CAA3

## 19 New absorbing conditions for FSI problems in blood flow

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**Keywords:** blood flow, absorbing boundary conditions

One of the main features of blood flow in the human arterial system is the existence of pressure waves originated by the interaction of blood with the vessel walls. We use a 3D fluid-structure interaction (FSI) model to capture the propagation of pressure waves, modelling the blood as a generalized newtonian fluid and the motion of the vessel wall by an hyperelastic non-linear model. In order to handle nonphysical reflexions related to the truncature of the physical domain, we analyze the use of different boundary conditions. We start by coupling the 3D FSI model with 1D hyperbolic models, known to properly capture the propagation of pressure waves, and derive energy estimations for the continuous coupled problem. Next, a set of boundary conditions derived from the 1D hyperbolic models is imposed directly to the 3D problem, and this new approach is compared to the previous coupling, using several numerical tests in idealized and real geometries.

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Session MLS4

## 20 Optimal control for some chemotherapy models

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**Keywords:**

The combination of mathematical modeling and optimal control theory provides a low-cost method for determining the most favourable treatment methods for cancer patients. In this talk, we present how to apply optimal control techniques to obtain the optimal schedule to reduce as much as possible the size of the tumor while maintaining tolerable levels of drug-induced toxicity. We will present theoretical and numerical results for several models based on free boundary problems for PDEs.

Session *MLS1*

## 21 Functional data analysis applied to environmental chlorophyll bio monitoring

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**Keywords:** chlorophyll, environmental monitoring, data analysis, functional data

Environmental analysis commonly works with large data sets, which creates several difficulties to study their patterns, where a random variable may assume an almost continuous range of discretizations increasingly extensive. These environmental measurements are available from a continuous atmospheric monitoring, typically obtained from satellite observations or collected by oceanographic ships. These data may then be used by adequate statistical models, which allow to predict environmental scenarios justified by sampled data.

Data measurements of smooth processes over time, like the ones we just described, may come from a process naturally described as functional. The observed curves are called "functional data" [3] and statistical methods for analyzing such data are described as "functional data analysis" (FDA). In this work, we apply FDA techniques, namely functional regression, to environmental bio-monitoring data, more specifically to satellite data of chlorophyll and sea surface temperature collected over time in Atlantic Ocean. We have available monthly data, from July of 2002 to October of 2010, collected over a grid in four regions repre-

senting areas of Atlantic Ocean with distinct characteristics. We aim to study the intensity of chlorophyll under the influence of temperature and the geographic region.

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Session *PSP3*

## 22 Integral bases in global fields

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**Keywords:** Dedekind domain, global field, local field, Newton polygon,  $p$ -integral bases, reduced bases

Let  $A$  be a Dedekind domain whose field of fractions  $K$  is a global field. Let  $\mathfrak{p}$  be a non-zero prime ideal of  $A$ , and  $K_{\mathfrak{p}}$  the completion of  $K$  at  $\mathfrak{p}$ . The theory of higher order Newton polygons developed in [3] and revised in [2] provides an algorithm that factorizes a monic irreducible separable polynomial  $f(x) \in A[x]$  over  $K_{\mathfrak{p}}$ , and it provides essential arithmetic information about the finite extensions of  $K_{\mathfrak{p}}$  determined by the different irreducible factors (cf. [3] and [1]). In particular, it can be used to compute a  $\mathfrak{p}$ -integral basis of the extension of  $K$  determined by  $f(x)$ .

In this talk we present a new and faster method to compute  $\mathfrak{p}$ -integral bases, based on the use of the quotients of certain divisions with remainder of  $f(x)$  that occur along the flow of the above algorithm.

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Session CAA4

## 23 Modeling multicellular growing systems

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**Keywords:** Living systems, kinetic theory, multicellular systems, active particles, hyperbolic limits, high-field regimes.

This work deals with the analysis of the asymptotic limit towards the derivation of hyperbolic macroscopic equations for a class of equations modeling complex multicellular systems. Cellular interactions generate both modification of biological functions and proliferating destructive events related to growth of tumor cells in competition with the immune system. The asymptotic analysis refers to the hyperbolic limit to show how the macroscopic tissue behavior can be described by linear and nonlinear hyperbolic systems which seems the most natural in this context.

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Session MLS3

## 24 Level Crossing Ordering of Markov and Semi-Markov Processes

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**Keywords:** birth-death processes, Markov processes, particle systems, queueing networks, sample-path approach, semi-Markov processes, stochastic ordering

Stochastic Ordering is an important area of Applied Probability that can be used effectively in qualitative comparisons of random variables, random vectors, and stochastic processes. In addition, it may be used to investigate the impact of parameter changes in important performance measures of stochastic systems, avoiding exact computation of those performance measures. The great diversity of performance measures used in applied sciences to characterize stochastic systems has led to the proposal of many types of stochastic orderings. In this talk we address the level crossing ordering, which was proposed by A. Irle and J. Gani in 2001 [1].

The level crossing ordering is an important and natural stochastic ordering that compares stochastic processes in terms of the times they take to reach high levels (states). More precisely, a process  $X$  is said to be slower in level crossing than  $Y$  if it takes  $X$  stochastically longer than  $Y$  to exceed any given level. In the original definition of Irle and Gani, the times the compared processes take to reach a given level or above are compared in the usual (in distribution) ordering sense, but other stochastic orderings can be used for the same effect. We work, in particular, with integral stochastic orderings weaker than the usual stochastic ordering, comprising many important stochastic orderings, such as the increasing convex ordering, widely used in finance and economics.

After introducing some motivation for the use of the level crossing ordering, we present tailored sufficient conditions that we have derived for the level crossing ordering of (univariate and multivariate) Markov and semi-Markov processes. These conditions are applied to the comparison of birth-and-death processes with catastrophes, queueing networks, and

particle systems. Our analysis highlights the benefits of properly using the sample path approach, which compares directly trajectories of the compared processes defined on a common probability space. This approach provides, as a by-product, direct algorithms for the simulation of stochastic processes ordered in the level crossing ordering sense. In the case of continuous Markov chains, we resort additionally to the powerful uniformization technique, which uniformizes the rates at which transitions take place in the processes being compared.

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Session P6

## 25 Cerebral aneurysms and mathematical modeling: from biological investigation to clinical application

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**Keywords:** Cerebral aneurysms, mechanotransduction, CFD and medical imaging

Cerebral aneurysms are pathological focal dilations of the cerebral arteries whose prevalence is estimated to be around 5% of the general population. The rupture of a cerebral aneurysm is frequently a catastrophic event, often resulting in death or severe disability, with less than half of all patients experiencing an aneurysmal rupture ever returning to the previous clinical status. One of the most important and recently debated features concerning brain aneurysms involves the natural history of incidentally diagnosed aneurysms. With the widespread availability of diagnostic brain imaging, more and more aneurysms are being diagnosed each year on asymptomatic patients, raising the issue to the level of a public health question. The main interrogation is related to how many and which of these aneurysms should be treated; if one realizes that the treatment modalities, whether by surgical or endovascular techniques, involve significant risks (both have

associated morbidity and mortality) and that this group of patients is mostly neurologically intact and has no symptoms, the need to have decision tools is evident. Up until now, the treatment algorithm has relied on the presence of the above mentioned risk factors for rupture, on the perceived treatment-associated risk (varies according to the center expertise) and on the data of the main prospective randomized study available, which is the ISUIA study. There is a pressing need for decision tools that take into account both mechanical, geometrical and biological factors (as well as their interplay), allowing us to predict the long-term evolution of the aneurysm and restricting treatment for those cases where it is indispensable.

In this presentation, we give an account of the potential usefulness of mathematical applications in the clinical setting, when dealing with brain aneurysms. We will focus on the possibility of using results of computational fluid dynamics as predictive tools, but also as a tool to unravel the phenomena of mechanobiological response of vascular wall cells to mechanical stress. Increased hemodynamic stresses (such as elevated wall shear stress) may trigger physiologic vascular remodeling as an adaptive process; somewhere along this process, this response becomes maladaptive and the aneurysmal dilation ensues. We hope to approach in the near future the study of this interplay by using real aneurysmal tissue in which phenomena such as extracellular matrix degradation, endothelial dysfunction, apoptosis and immunoinflammatory response can be measured and correlated to mathematical models of blood flow and rheology.

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Session MLS2

## 26 Study of the density for semilinear dissipative parabolic SPDEs

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**Keywords:** Stochastic partial differential equation, existence and smoothness of densities, Malliavin calculus.

We prove existence and smoothness of the density of the solution to a semilinear parabolic SPDE on  $L^2(\mathcal{O})$  (evaluated at fixed  $(t, x) \in (0, T] \times \mathcal{O}$ ) driven by an additive cylindrical Wiener noise with dissipative drift of polynomial growth, where  $\mathcal{O}$  is a bounded domain in  $\mathbf{R}^d$  with smooth boundary. The proof uses the Yosida approximation of the drift and tools of Malliavin calculus.

Session PSP4

## 27 Food processing by high pressure technologies. A mathematical approach

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**Keywords:** Modeling, simulation, high pressure, food industry

Nowadays, in industrialized countries, food products are processed in order to prolong their shelf life and to maintain or even improve their natural qualities such as flavor and color. Decomposition of food is mainly due to microorganisms and enzymes. Consumers look for additive-free food products that maintain their organoleptic properties. This has promoted the development of new technologies for food processing. One emerging technology is High Hydrostatic Pressure, as it has turned out to be very effective in prolonging the shelf life of foods without losing its properties. Other applications of high-pressure technology on food processing, are the ones that deal with ice-

water transitions: high-pressure freezing, high-pressure thawing and preservation in a non-frozen state at sub-zero temperatures. High- Pressure Shift Freezing (in which phase transition occurs as a result of a change of pressure) is particularly interesting and advantageous compared to classical freezing, because supercooling occurs after the pressure release, throughout the whole volume and not only on the surface. Therefore, small granular shaped ice crystals are formed and distributed homogeneously throughout the sample. We will explain different ways of approaching the mathematical modeling of these processes and we will show some results.

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Session MLS4

## 28 Automated Deduction in Geometry through Symbolic Computation

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**Keywords:** automatic theorem proving, automatic discovery in geometry, computational algebraic geometry, symbolic computation

The purpose of this talk is to present, to non-specialists in the subject, basic results and problems, as well as some recent developments by this lecturer, from automatic theorem proving and discovery in elementary geometry, by means of computational algebraic geometry tools. It should be mentioned that initial attempts to implement automatized theorem proving in geometry appear already, in the realm of Artificial Intelligence, in the 50's. An early and relevant contribution in this direction is the work during the 70's of the Portuguese professor Luís Moniz Pereira (with Helder Coelho) on GEOM, a Prolog-based geometry theorem-prover. On the other hand, the algebraic geometry approach to this issue was also started in the 70's by professor Wen-tsun Wu, who received, in 2006, the Shaw Prize (jointly with D. Mumford) for his contribution to the mechanization of mathematics.

Very roughly speaking, the goal is to assert, algorithmically, the truth/falsity of a geometric statement of the kind  $\{H \Rightarrow T\}$ . And, if declared false, to find complementary hypotheses,  $H'$  such that  $\{(H \wedge H') \Rightarrow T\}$ . We refer to [2], [6], for details about the automatic proving and discovering method. The underlying tools come from symbolic computation and algorithmic elimination theory, such as Gröbner basis or, more recently, Gröbner Covers, specially well suited for automatic case distinction (see [4]). Given the current powerful performance of these tools, many involved classical problems can be automatically addressed, with spectacular achievements (see, for instance, the generalization of the theorem of Steiner-Lehmus in [3]).

An obvious field of application for these techniques is mathematics education. A desirable goal –with a potentially broad impact at school– is to provide Dynamic Geometry (DG) programs with automatic proving and discovery features (see, for instance, the title of the paper of our Portuguese colleague P. Quaresma in [5]). The pioneer work of F. Botana, developing the program GDI (Geometría Dinámica Inteligente), is a reference

on this issue (cf. [1]). We are currently working together, as well as with GeoGebra developers (the most popular DG at the moment), towards this goal.

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Session P1

## 29 A functional implementation of rational languages equivalence, using bisimulation

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**Keywords:** Rational Languages, Rational Expressions, Equivalence, Bisimulation, Funcional Programming.

The classical way to test if two rational expressions are equivalent, i.e. if their represented languages are the same, is to convert each to an equivalent non-deterministic finite automaton; then convert this automaton into a deterministic one; then obtain the minimal equivalent automaton, that, because of its uniqueness, can test of the equality of the languages. This long process can be avoided through the use of Brzozowski's derivatives of rational expressions. This method, that corresponds to test of the existence of a



bisimulation between the two rational expressions, have a particularly efficient and elegant implementation in a functional language like Haskell.

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Session CAA2

## 30 Algebraic Topology: formalization and applications

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**Keywords:** formalization, Algebraic Topology, biomedical image processing

In the talk, we will report on the developments carried out in Isabelle/HOL, ACL2 and Coq/SSReflect on Algebraic Topology, in the frame of the ForMath european project. Special attention will be paid to the applications in biomedical image processing.

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Session CAA1

## 31 Mathematics of blood coagulation

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**Keywords:**

Mathematical modeling of blood coagulation provides a good example of the interaction between mathematics and life sciences. The most complete

models consist of a system of convection-reaction-diffusion partial differential equations, for the biochemical species, coupled with the Navier-Stokes equations, or some other non-Newtonian model, for the blood flow and its rheology, [1]. The evolution of such a system depends on the action of several blood Factors, which regulate the chemical reactions, as well as on mechanical factors, [3]. Besides, the existence of a high number of dependent variables, and the existence of several space and time scalings, [5], make it difficult obtaining an analytical solution. An overview of this modeling problem is presented including some stability results, [4], and numerical simulations, [2].

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Session MLS1

## 32 Integer-valued Asymmetric Power ARCH Processes

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**Keywords:** Volatility, integer-valued time series, APARCH models

Most models for financial time series are given in the form  $X_t = \sigma_t Z_t$ ,  $t \in \mathbb{Z}$ , where  $(Z_t)$  is an i.i.d. zero-mean sequence with unit variance and  $(\sigma_t)$  is a

stochastic process such that  $\sigma_t$  and  $Z_t$  are independent for fixed  $t$ . This representation somehow expresses the belief that when analyzing financial time series like price changes or stock index fluctuations, the direction of the changes cannot be modeled or predicted, only their magnitude. This magnitude is related to  $\sigma_t$ , which is commonly called the volatility of the process. Bollerslev (1986) introduced the Generalized ARCH( $p, q$ ), where

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i X_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

with  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$ ,  $i = 1, \dots, p$ ,  $j = 1, \dots, q$ . The family of GARCH models have been generalized and extended in various directions to accommodate some common feature often exhibited by financial time series. One extension was proposed by Ding et al. (1993) who introduced the Asymmetric Power ARCH or, in short, APARCH( $p, q$ )

$$\sigma_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|X_{t-i}| - \gamma_i X_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta$$

with  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$ ,  $\delta \geq 0$  and  $-1 < \gamma_i < 1$ . This model allows to detect asymmetric responses of the volatility for positive or negative shocks. If  $\gamma_i > 0$ , negative shocks have stronger impact on volatility than positive shocks, as would be expected in the analysis of financial time series. If  $\gamma_i < 0$ , the reverse happens.

In this talk, we introduced the Integer-valued APARCH( $p, q$ ) model which can be considered as the discrete counterpart of the conventional APARCH( $p, q$ ) model and is suitable for modeling time series of counts. The Integer-valued APARCH( $p, q$ ) is defined to be an integer-valued process  $(X_t)$  such that

$$\begin{cases} X_t | \mathcal{F}_{t-1} \sim Po(\lambda_t), \forall t \in \mathbb{Z} \\ \lambda_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|X_{t-i} - \lambda_{t-i}| - \gamma_i (X_{t-i} - \lambda_{t-i}))^\delta + \sum_{j=1}^q \beta_j \lambda_{t-j}^\delta, \end{cases}$$

where  $\mathcal{F}_{t-1} = \sigma(X_s, s \leq t-1)$ . Basic probabilistic and statistical properties of the Integer-valued APARCH(1, 1) model will be discussed. Moreover, parameter estimation will be also addressed.

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Session PSP4

## 33 Optimal Parametrizations of Algebraic Curves and Surfaces

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**Keywords:** Algebraic curve, algebraic surface, parametrization, optimality criteria.

An algebraic variety, or more particularly an algebraic curve or surface, can always be represented by means of implicit equations. Moreover, these implicit equations can be chosen such that, under certain criteria, are unique. On the other hand, under certain hypotheses, the algebraic curve (resp. surface) can be represented by means of rational functions; such a representation is called a rational parametrization. Nevertheless, when this is possible, there are infinitely many different parametric representations. In order to choose an *optimal* parametrization of the variety one may consider different criteria based, for instance, on the injectivity or on the surjectivity of the rational map induced by the rational parametrization, or on the simplicity of the coefficients involved in the rational functions of the parametrization (here one requires to express the parametrization over the smallest possible field extension of the ground field), etc.

In this talk, after introducing the main concepts and motivating the ideas above, we plan to review some algorithmic methods to either parametrize or reparametrize algebraic curves and surfaces under different optimality criteria. More precisely, we also plan to focus on some algorithmic questions when the ground field is the field of the real numbers but the information is given over the field of the complex numbers; for both cases, curves and surfaces.

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Session CAA3

## 34 Mathematical model of eye pathologies related with the porosity of tissues

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**Keywords:**

Glaucoma is an eye disease in which the optic nerve is damaged and it is the first cause of blindness in the USA and Europe. In most cases is characterized by an high intraocular pressure in the anterior chamber of the eye, induced by occlusion of a porous structure - the trabecular mesh (TM) - that prevents the normal dynamic of the flow in the eye - the aqueous humor (AH). In this talk a mathematical model that simulates the lowering of IOP, when drugs that act on the trabecular mesh are released into the cornea allowing an increase flow of AH, is presented. The model consists of a set of coupled systems of PDEs. Considering clinical data that relates the change in the structure of TM, as being the cause of the obstruction, particular attention will be devoted to the influence of porosity and permeability in the outflow of AH. We will present numerical simulations that provide interesting clinical information.

Session *MLS3*

### 35 Pattern formation in a flux limited reaction-diffusion equation of porous media type modeling morphogenetic action

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**Keywords:** Flux limited, Porous media, Relativistic heat equation, Pattern formation, Singular traveling waves, Non-linear reaction-diffusion, Optimal mass transportation, Entropy solutions, Complex systems, Morphogenesis

In this talk, motivated by various experiments in biology (morphogenesis and propagation of tumors), we analyze the mathematical properties concerning pattern formation in new models that combine the properties of flux limiters along with the porous media systems.

On the one hand, a nonlinear PDE featuring flux limitation effects together with those of the porous media equation is introduced. We analyze the balance of such diverse effects through the study of the existence and qualitative behavior of some admissible patterns to this singular reaction–diffusion equation. We show the existence of different types of traveling waves: classical profiles for high enough wave speeds, and discontinuous waves that are reminiscent of hyperbolic shock waves when the wave speed lowers below a certain threshold. Some of these solutions are of particular relevance as they provide models by which the whole solution (and not just the bulk, as it is the case with classical traveling waves) spreads through with finite speed.

On the other hand, a central question in biology is how secreted morphogens act to induce different cellular responses within a group of cells in a concentration-dependent manner. Modeling morphogenetic output in multicellular systems has so far employed linear diffusion. However, such models necessarily imply unrealistic instantaneous spreading of morphogen molecules which is induced by the assumptions of Brownian motion in the continuous formulation. To remove these non-biological behaviors we introduce a novel approach, based on the previous study on pattern formation as well as on biological experiments, that assumes restricted velocity of propagation of the morphogen signal (focusing on Hedgehog-Gli signaling), and thus non-instantaneous diffusion, by applying flux-limited spreading. The new models predict concentration fronts and the evolution of gradient dynamics and responses over time.

The talk is based on several collaborations, of which the following are only a few references:

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Session P5

## 36 Eigenvalue computations in the context of data-sparse approximations of integral operators

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**Keywords:** Iterative eigensolvers, integral operators, projection approximation, hierarchical matrices

We consider the numerical solution of a large eigenvalue problem resulting from a finite rank discretization of an integral operator. We are interested in computing a few eigenpairs, with an iterative method, so a matrix representation that allows for fast matrix-vector products is required. Hierarchical matrices are appropriate for this setting, and also provide cheap LU de-

compositions required in the spectral transformation technique. We illustrate the use of freely available software tools to address the problem, in particular SLEPc for the eigensolvers and HLib for the construction of H-matrices. We develop analytical expressions for the approximate degenerate kernels and deduce error upper bounds for these approximations. Numerical tests show the benefits of the data-sparse representation compared to standard storage schemes, in terms of computational cost as well as memory requirements.

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Session CAA3

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[Abstract 36; Session CAA3]